Mathlets in Group Work, Segment 1

Mathlets: An Introduction

HAYNES MILLER: One of the most exciting and educationally powerful uses of Mathlets is in structured group work. The first segment of this module is another video fragment from the MIT workshop. This video illustrates the kind of thing you can ask students to do fairly quickly in a lecture setting. As usual, each segment will be followed by some discussion, and at the end of the module there are some exercises for you.

My objectives in this first segment are these. I want to exemplify the use of Mathlets in setting up group work in lecture and show how to encourage the use of Mathlets to stimulate mathematical reasoning by students.

I've got isoclines up here. Isoclines is another concept from differential equations. But again, I think that we can learn a lot without knowing the technical details of how this works. And many of these applets, including this one, have a menu here.

And I'm going to select from that menu not the default equation, but rather the equation y prime equals y squared minus x. That brings up a slightly different picture. And I'm going to set it-- down here in the bottom there's a button that says slope field which I can set here. And this one thing we do for a display. When I have it in this setting, the display of the direction of the slope field is very clear.

OK, so here's this picture, which looks to me like a hayfield that a wind storm has gone over or something and knocked down the hay. If you're up on differential equations, it's a slope field. It's a representation of the differential equation y prime equals y squared minus x. But if you're not up on differential equations, It's a piece of art. It looks to me like a hay field, and the hay has been knocked down in this pattern.

And what we're interested in are deer tracks through that hay field. So if you squint, you can sort of see the way deer would like to run through that hay field, leaving one shock of hay on one side and another piece of hay on the other side. You can sort of see what they must look like, what those trajectories of deer, what those deer tracks must look like.

So if you squint hard enough, I can make them appear before your eyes by clicking here. There. There's the deer, running along. And see, he's always leaving these little direction elements, these little line segments tangent to his motion. So there's a lot of them. A lot of possible curves that have that behavior. You can do this too. I'm just dragging the cursor across the field. And you can see all these different possible paths that the deer will take. So I can drop one down there and just leave it. There's one possible deer path.

There's a word-- in the trade, these things are called integral curves. They are graphs of solutions of this differential equation. And I can click up. I see a bunch of screens already. You've already discovered that you can click up a lot of different solutions here, a lot of different integral curves.

OK, so the fun here is looking at what this says about solutions. This is a non-linear differential equation. You've drawn the graphs of functions that you've never seen before. These functions cannot be written in terms of elementary functions. You cannot write them in terms of exponentials and sines and cosines and stuff like that. They're just new in the world. They're maybe new to you. They're certainly new to your students.

They're functions that we can study. And this particular differential equation models various things. So these functions will speak to the behavior of some system.

But we can ask questions about these new functions, like, here's a question. Some of these functions seem to have maxima or minima, and some of them seem not to. So here's a question for you. Where do the maxima, where do the extreme points occur?

Just from looking at it, you can sort of see that this solution has a maximum and this solution has a maximum. These seem to have maxima down here. But then the solutions over here don't have maxima. So where do they occur? If I have a solution, can I-- I don't know what the function is. I don't have a way to write it in terms of sines and cosines or something. But I can still ask the question and maybe even answer the question of where the maxima and minima occur.

So work on this for a minute. I'll let you talk about it for a minute, and then I'll try to get some answers from you.

PARTICIPANT: [INAUDIBLE] parabolic.

PARTICIPANT: Half negative slope is this way. Positive slope is that way.

PARTICIPANT: And so--

PARTICIPANT: This is positive slope--

PARTICIPANT: --positive slope would be that way, and negative-- yeah. Yeah. So that's--

PARTICIPANT: [INAUDIBLE] and actually, nice sharp screen.

PARTICIPANT: [INAUDIBLE]

PARTICIPANT: [INAUDIBLE] Have a prediction?

HAYNES MILLER: OK. That's right. So where does that match? Slope here is horizontal. There's a formula for the slope here. y prime is given by that formula.

PARTICIPANT: So if x equals y squared--

HAYNES MILLER: And what shape is that?

PARTICIPANT: I don't--

PARTICIPANT: Parabola. Horizontal parabola.

HAYNES MILLER: OK, and that looks pretty good, doesn't it?

PARTICIPANT: Yeah.

HAYNES MILLER: OK. So now I can show you something that I'll show at large here.

PARTICIPANT: Good job answering your own questions.

HAYNES MILLER: OK, are there some predictions? Some thoughts about where that maxima occurs?

PARTICIPANT: Where your slope is 0.

HAYNES MILLER: Where did the maxima occur?

PARTICIPANT: At the [INAUDIBLE]--

PARTICIPANT: Where the slope is 0?

PARTICIPANT: Where-- the point--

HAYNES MILLER: Where the slope is a horizontal, that's where maxim are going to occur. And the formula says that y prime is y squared minus x. That happens when y squared minus x is 0, which is a parabola on its side opening out to the right. And that does look like where the maxima are occurring, that parabola.

Now, if you squint really hard, I can make that parabola appear. There it is. Did you see that happen? It's a little hard to see on the screen. But if you look now, it's there. It's in yellow on the screen. So what I did, there's a slider up here called m. And I selected m equals 0. You can do that too.

PARTICIPANT: Click on that. Yeah, you click on that.

PARTICIPANT: [INAUDIBLE]

HAYNES MILLER: So that yellow curve is the place where the slope field is horizontal, or the slope of the field is 0. And that's the place where the maxima and minima will occur. That's called an isocline. Iso means same. Cline means slope. It's the place where the slope of the slope field is constant, 0 in this place. It's called a null cline because the slope is 0.

I can draw other isoclines also, like here's the m equals 1 isocline. Well, let's try the m equals 2 isocline. There it is. It's another parabola. That's y squared minus x equals 2. It's that parabola. And everywhere under that yellow curve that I just drew, this yellow parabola, the slope of the curves that go through it is 2, just the way everywhere under this one the slopes are 0. So you can draw more eyes of clients and get more of a feel for what these functions are looking like.

But I have another question for you. These look like they're maxima. They look like all the critical points, all the extreme points that occur in maxima. Can you see, can you verify, can you justify that? Can you explain why they should all be maxima?

PARTICIPANT: When the slope is 0 of the tangent, that's the maximum, right?

HAYNES MILLER: Yes. Although it might be a minimum.

PARTICIPANT: It might be a minimum. That's true.

PARTICIPANT: It might be a minimum.

HAYNES MILLER: So why is it not a minimum? That's the question.

PARTICIPANT: Oh. If the slope before that point, you were asking if it's a maximum?

HAYNES MILLER: Yeah. Why is it a maximum?

PARTICIPANT: The slope before that point to the left is positive and then 0 and then negative.

HAYNES MILLER: Ah. Great. And I can see that. I'm going to come back here and drag this out. So I'm looking at the isoclines for positive slope, they are parabolas to the left of the null cline. And over here, these have negative slope to the right. So when a solution passes through the null cline, just before it hits the null cline it has positive slope, just after it has negative slope, and so it must be-- going up and then coming down, it must be a maximum.

That's actually a proof. This is a mathematical argument that students can do without effort. It's not hard. It's Rewarding to verify what you can observe on the screen here. You can do it with calculus too. I mean, I could calculate the second derivative at that critical point and find out that it's negative. And that's another direction that you might want students to go in.