Mathlets in Lecture: Segment 2

Mathlets: An Introduction

HAYNES MILLER: The second segment of this module models a more interactive lecture style. Because my audience was a group of science teachers, I chose a Mathlet illustrating some rather basic mathematical concepts. Here's a slide capturing my objectives in showing you this recording. These are the same as my objectives were in the original presentation. The lecture fragment objectives were, the participant will be able to use Mathlets to support lecture-based instruction, to increase student participation in the lecture, to use the Mathlets to give students a visual correlate for symbols and calculations.

I'm going to look at graph features. So I think graphs of functions are common language in science. And so I'm going to ask everybody to play along with this rather more simple tool that's written for a calculus course.

So what you see is a graph. It's a graph of a third-order polynomial. The polynomial is described here. It's x cubed minus and so on. And these coefficients, 1, minus 2, 0.5, and 1, they're controlled by these four sliders. So a is the coefficient of x cubed. And b is the coefficient of x squared minus 2. It's set at minus 2 right now.

Part of the purpose of this tool is to help students get the idea of functions. And so by moving this slider here, I can change the value of x. And the student can see the output of the function and get the idea that a function is a rule that takes an input and produces an output. But after a while, you get more interested in the global features of this function. And then that evaluation utility becomes kind of annoying. And I'm just going to set x equal to 0 now and look at the rest of what we see.

So there are other features that the students should be learning about in a pre-calculus course. You may be interested in showing them where the function is rising or where it's falling, a little section there where it's falling in green. And you can see, again, the color coding that happens.

So rising and falling are simple things. Students have a harder time with concepts like concave and convex. Let's see. Concave has to do with a cave, so it curves down like this. But it includes places where the function is increasing and decreasing. It has nothing to do with the issue of increasing and decreasing. This is a hard thing for students to get sometimes. And there's the convex portion.

Well, right now this function zooms up as x gets to be large. How would I change the coefficients to make it zoom down when x gets to be large instead?

PARTICIPANT: The first coefficient is negative.

HAYNES MILLER: All right. So the first coefficient is this a. It's set at one right now. Let's experiment. There's a proposal from the floor. Let's find out. We can do an experiment here and see what happens. Very interesting. I'm not sure you've ever really visualized what happens when you start changing these coefficients.

Well, it disappears off the screen. Now it's become negative. Ah! And there we go. Now it's coming down from plus infinity to the left and then sinking down to minus infinity on the right. So that leading coefficient is what determines that particular feature.

Now, look, we could continue to play with complicated functions like cubics, but let's go in the other direction and simplify this a whole lot. I'm going to set a to be equal to 0. So I've gotten rid of the cubic term. And actually, it disappears in the display down here. And just for a start, I'm going to get rid of the quadratic term as well.

So now this is just a first-order linear if you like, a first-order polynomial equation. Because the graph is a straight line, of course. And these two parameters are called the slope. There it is. I'm changing the slope and the intercept. And I can move the intercept up and down. So this is helpful for students. And maybe I can make it brighter by saying that it's rising.

So let's make it a little more complicated now. I'm going to reintroduce the quadratic term here. And there we go. So now I have a parabola, right? The graph of a quadratic equation is a parabola. So I have some questions about this parabola. The parabola doesn't seem to go through 0 right now. How can I make the value of this function-- how can I make the graph go through the origin?

STUDENT: Translate down.

HAYNES MILLER: OK, so--

PARTICIPANT: Put b equals 0.

HAYNES MILLER: If I set x equal to 0, then what's left is the constant term. So I guess I'll set d. I can move d. d just slides the graph up and down. Ha. All right. So I'll set d to be 0, and there we go. Now the graph goes through the origin.

Now how about this b coefficient? So I have a question for you here. So what will happen if I increase that b, the coefficient of x squared, in that formula? Will the graph, will the parabola get thinner? Or will move without changing its shape? Or will it get fatter? I mean, increasing b, will it make the parabola fatter?

Participants: Thinner.

HAYNES MILLER: Should we take a vote? How many of you say thinner?

PARTICIPANT: Thinner.

HAYNES MILLER: How many of you say fatter? See now, I saw some virtual votes there. This is one of the great reasons for not using public voting, but using private voting. In a big lecture hall-- I wish I'd brought them in-- I use, I call them flash cards. I use numbered cards that have numbers like 1 through 8 on them. And students will vote by putting up these numbers. And it's fantastic, because I teach in a big lecture hall. I teach this course in a big lecture hall. And I see these numbers in the hall. And the students do not. The students do not look around to see how other people are voting. They respect the privacy of their peers. And it's very genuine. You get a big scatter if you ask a question. You get a big scatter of different answers, which then leads to discussion as well. All right. So we didn't get-- so I think the winner here in the election was-- thinner?

PARTICIPANT: Yeah.

HAYNES MILLER: Thinner. If I increase b, it's supposed to get thinner. There we go. And, of course, if I make b negative, then I get a parabola that opens down. All right. So let's put b up to some positive number. And I'm going to make d look more interesting as well.

All right. So now I have another question for you. What role does the coefficient of the linear term play? If I change that linear coefficient, which is 0 right now, if I increase it, what will happen to the parabola? So again, I'm going to come back to my--

PARTICIPANT: The vertex follows that... .

HAYNES MILLER: So here's my question. c is the coefficient of the linear term, that x. So the parabola I'm looking at has the formula y is bx squared plus cx plus d. If I increase c, what will happen to the parabola? Now this is a pretty tough question.

PARTICIPANT: It goes left, ... I don't know.

HAYNES MILLER: So this is the kind of question that really calls for-- I mean, this would be an unfair question, I think, to a class, unless you want them to sit down and do a calculation-- which I could ask you to do right now. But-- I tell you what. I'll do it with you. And then we could take a vote now, but I don't think it's really fair, because you have to do a calculation, really, to come up with an answer.

So let's go back to the slide here. So what really is at issue is we want to know how this vertex moves. Let's find out how that vertex moves. Let's find out the x-coordinate of that vertex.

So here's a calculation of the x-coordinate of that vertex. So I'm going to call x of b, the horizontal, the x-coordinate of the bottom of the parabola, right? And you can find that by setting the derivative equal to 0. Or you can just do it with algebra. And that's called completing the square.

So if I complete the square, that's this algebraic construction that gets rid of the middle term and writes it inside of the square of something. And if you think about it, it's got to have this form. x plus c over 2b is the right thing to put there. Because when you FOIL it out, it comes out with the cx in the middle.

All right. Now where does the vertex occur? Well, it's going to occur when this term is 0, right? That's when the minimum happens, when this term is 0. That means that x of v is minus c over 2b.

Minus c over 2b. That's a really interesting formula. So it says a lot. If we have b fixed and we move c, I said increase c, what's going to happen to x of v? It's going to decrease. It's going to move left. Somebody said move left. I think they knew something here.

So let's go back to the screen and see what happens if I increase c. What's going to happen? The prediction is it's going to move to the left. So if we move c to the right, the prediction is the vertex is going to move left. Yay! Look at that. All right. So that's because it's a minus c over 2b, right?

Now, let's go back to that formula again. So there's c over 2b. That b, the coefficient of the x squared, comes into this formula too. And this says that the vertex should move a lot more if b is small. Right? If the denominator is small there, then x of v will change more when I change c.

Let's test that out. So I'm going to make b smaller. That'll make the parabola fatter, right? Yeah. OK. And now I'm going to change c. And the prediction is it's going to move much more. And it does. It moves much more dramatically. Because there's that b in the denominator for the formula.

In fact, watch this. Watch what happens to that vertex when I-- so here's b. I'm going to really play with b. I'm going to send b through 0. So then I get a straight line. There's no vertex. That vertex is going down to the left. It's going to go through infinity, out the other side, and come back on the other side. See that? It goes right through infinity and comes back on the other side.

So now, that was quite a bit of-- so now, look. This is a polynomial. This is parabolas, right? I don't think I could've interested you even as much as I did in the behavior of parabolas without the assistance of this kind of technology. The graphic nature of it is a big help in getting people's attention and letting them think about it.

And this calculation I did, this is a fair bit of algebra. There's no numbers in here except for 2. This is all symbols. But because you can see the way they correlate with things on the screen here, the meaning of those letters is more transparent than if you're just giving an abstract fraction. So that combination of seeing things visually and then doing an analysis, it's a very powerful combination.

What did you think about that lecture fragment? Once again, I began slowly, introducing the various features visible on the screen one by one. This method is considerably simpler than the first lecture fragment, so the exploration completed more quickly.

Here are some questions for you to think about. This was an interactive lecture with quite a few suggestions and answers from the audience. How many students do you think provided that feedback? Do you think I had any information about how much the other students were understanding? Second question. What further lessons could you draw from this Mathlet in a lecture setting?

Please refer to the course website for a written discussion of the lecture fragment that we have just seen.